Abstract - This article is intended to introduce the Mathematical Learning Model of APOS Theory (Computer Based APOS Model). The APOS model has been developed using The Plomp design consist of three initial phase: 1) preliminary research, 2) prototyping phase, and 3) assessment phase. The construction of the model used the Joyce and Weil models, which consist of five components: Syntax, Social System, Reaction Principle, Supporting System and Impact. The syntax of APOS model consists of phases: Orientation, Practicum, Group Discussion, Class Discussion, Exercise, and Evaluation. The developed APOS model was valid, practical and effective already. The main supporting part of the APOS Model were Worksheets that contain activities in the Orientation phase, Practicum phase, Group Discussion phase, and classroom discussion phase. For Evaluation Phase we used separated instruments. APOS model was being implemented in Integral Calculus Course by Mathematics Education Students Semester 3 FKIP UNIB FY 2017/2018. In the Practicum phase, students work with computers using the Maple program for Calculus. Information collected through a postest for the Area of Polygon and Sums of Riemann topics as follows: The number of students who scored ≥ 80 was 33.33%. The number of students who scored ≥ 60 and <80 were 33.33%. The number of students who scored <60 was 33.33%. Information collected through an open questionnaire about which phase did the students understand the materials as follows: the number of students can understand the material in phase: Praktikum = 22.22%, in Small Group Discussion = 30, 56%, in Classroom Discussion phase = 33, 33%, and in Exercise phase = 13, 09%. It can be conclude that the model is good for improving student learning outcomes, and to improve students' ability in discussion.

Keywords: APOS Model, Syntax, Practicum Phase, Worksheet

I. INTRODUCTION

Information and communication technologies (ICT) is a sum of technology tools and resources for creating, spreading, changing and managing the information. ICT in education has brought many novelties and essentially changed its values, methods and results. That would be an important benefit for education system if the novelties and changes brought by ICT are accepted, used and developed by students, teachers, school administration, decision makers in local and state institutions and parents. With coming of ICT, our attention should be paid to paradigm of modern pedagogy – student is in the centre of practical educational process, they can learn independently in suitable place, time and speed (Cunska, Aija & Savicka, Inga, 2012).

Learning in the context of the Curriculum 2013 is oriented to produce a productive, creative, innovative, and affective Indonesian people through strengthening the integration of attitudes (know-why), skill (know-how), and knowledge (know-what). This orientation is based on the awareness that the development of life and science of the 21st century, has been change in characteristics compared with the previous century. A number of the characteristics of the 21st century is that this century is the era of information, computing, automation, and communication. This is what is anticipated in the 2013 curriculum (Abdin, Yunus, 2016).

Related to the the 21st century' eature as the era of information, computing, automation, and communication, learning in the twenty-first century must be developed using appropriate instructional design. In relation to the characteristics of information, the learning which should be developed is a learning directed to encourage learners to “find out” instead of the learning that make learners “get informed”. Learning that is encouraging students to “find out” is an active and constructivist learning. Therefore, learning in the 21st century should be designed on the basis of constructivist contextual learning approaches (Abidin, Yunus, 2016).

Changes brought by the integration of ICT in the teaching and learning of mathematics can be seen at two levels: ICT for learning mathematics, and new strategy for teaching mathematics with ICT. When a teacher uses ICT for learning mathematics, s/he can still design the teaching in an oldfashion way of face-to-face classical teaching, thus ICT is a mere technological tool which can assist students to learn, to do drill and practices, to do exercises on certain concepts in mathematics. In this context, As Pustari (2014) states, mathematical softwares have been the most ICT used in the teaching and learning of mathematics, e.g., Geogebra, Autograph, Maple, Mathematica, MathLab, WolframAlpha, Desmos graphing calculator, Microsoft Mathematics, etc., whether it is a free software or paid ones (Panen, Paulina, 2014).

Computer-based learning program is a learning program used in the learning process by using computer software in the form of computer programs that contain the subject matter in the form of exercises. It is in line with the statement by Robert Heinich, Molenda and James D. Rusel in Rusman (2014) which said that: "Computer system can deliver the instruction by allowing them to interact with the
lesson programmed into the system; this is referred to as computer-based instruction.

When designed properly, there are numerous potentials uses of ICT in education. As an integrated component of teaching and learning, ICT allows learning experiences which are innovative, accelerated, enriched, and deepened the skills acquisition, motivating and engaging and relating school experience to work practices and authentic context. The integration of ICT can help teachers and students to embark on the student-centered learning instead of content-oriented or teachercentered learning. Microsoft (2014) elaborates that ICT use in teaching and learning is potential at developing self-regulation, collaboration, knowledge construction, real-world problem-solving and innovation, and skilled communication. These potentials uses of ICT in education indicate that the simple use of ICT as a tool in teaching and learning is no longer adequate, and thus creative and innovative strategy on the use of ICT is called for (Panen, Paulina. 2014).

Starting from the problems encountered in learning mathematics, especially Calculus, as well as the importance of mathematics to assist students in developing high-level thinking, also the importance of innovation in student-centered mathematics learning, it has developed a Model Learning Calculus Based on APOS Theory (MPK-APOS), which is valid, practical and effective (Hanifah, 2015). When MPK-APOS is implemented in other mathematics courses, such as when applied to the Geometry Transformation course with the GeoGebra application program, the MPK-APOS name becomes inappropriate because the letter K stands for Calculus. Since the term MPK-APOS provides a narrow space, the name MPK-APOS is developed into Mathematical Learning Model Based on APOS Theory (Model APOS) (Hanifah, 2016).

APOS Theory is a learning theory devoted to mathematics learning at the college level, which integrates the use of computers, studying in small groups, and taking into account the mental constructs undertaken by students in understanding a mathematical concept. These mental constructions are: action, process, object (object), and schema (schema) which is abbreviated as APOS (Dubinsky, 2001; Arnawa, 2009).

A student can construct a mathematical concept well if the student experiences an action, a process, an object, and a scheme. A student is said to have had an action, if the student focuses his mind in an attempt to understand the mathematical concepts he faces. A student is said to have had a process, if his thinking is limited to the mathematical concepts he faces and is characterized by the emergence of the ability to discuss the mathematical concepts. A student is said to have possessed an object, if the student has been able to explain the properties of the mathematical concept. A student is said to have had a scheme, if the student has been able to construct examples of mathematical concepts in accordance with predetermined requirements. Therefore, the learning steps based on the theory of APOS according to Suryadi (2010) are as follows: (1) At the beginning of learning, lecturers should encourage students to perform activities in analyze issues related to the concept to be provided by using concepts that have been owned by students so that the minds of students will focus on the mathematical concepts he studied. This activity will trigger the students to have an action, (2) When the learning process takes place, the lecturer should act as facilitator and give indirect instruction so that the students are encouraged to do the math concept discussion more deeply and more generally. This activity will trigger students to have the process of mathematical concepts. Furthermore, if required, the lecturer should conduct intervention indirectly so that students can find or synthesize the properties of mathematical concepts. This activity will trigger students to have objects of mathematical concepts and, (3) At the end of the lesson, the lecturer should give the task of applying the concept and the task of constructing examples of mathematical concepts that meet certain requirements. This activity will trigger students to have a mathematical concept scheme (Suryadi, 2010).

Based on that theory, the general results of the development of the learning model that refers to the Plomp’s Model development (2013), and the using of Joyce and Weil’s (1992) Model construction, and after improving Mathematical Learning Model Based on APOS Theory (APOS Model) with the following components: (1) Syntax consists of: Orientation phase, Practicum phase, Small Group Discussion phase, Classroom Discussion phase, Exercise phase, and Evaluation phase. (2) Social Principles that occur when APOS Model is implemented is the cooperation between students, the scaffolding (assistance) from lecturer to student or from student to student. The existence of multi-direction communication from lecturer to student and/or from student to student. (3) Reaction Principle that happened is student centered learning, lecturer acting as supervisor, and assessment done is process assessment, not only from test result only. (4) The most important supporting systems are computers and worksheets with APOS, SAP / RPP models, syllabus, curriculum, math application programs, and guides. (5) Instructional impact that occurs is more students absorption, not easy to forget because students construct their own material. As a Companion Impact, students are active, confident, positive, motivated, and caring.

II. THE IMPLEMENTATION OF APOS MODEL

APOS model is currently implemented in the Integral Calculus course for Student of Mathematics Education Program Semester 3 FKIP UNIB FY 2017/2018, which was attended by 38 students, with 4 (3-1) credits in 4 x 50 minute implementation time, starting from at 8:00 ends at 12:00, which is
interspersed with a 20 minute break. One credit of practicum was usually carried out separately in the computer lab, after the teaching materials are taught by the lecturers in the class. In learning APOS model, lectures and practicum are implemented in class and the time is combined into one time that is 4 x 50 minutes = 200 minutes. The time allocated for each phase was as follows: 15 'Orientation phase, 50' Practicum phase, 50 'Small Group Discussion phase, 20' rest, 50' Classroom Discussion Phase, and 15' Exercise phase. Students study in a heterogeneous group consisting of 3 people per group. Each group carried out minimal one Laptop. In the Practicum Phase students work using Laptops with Maple application program.

To support the implementation of the APOS Model, a Calculus Worksheet was developed which included activities for each phase. In the Practicum phase, the Worksheet is designed containing the Maple commands to be executed by the computer. The selected Maple command follows the step of solving a problem manually. So when the command is executed, the result is able to explain the concept of a calculus material. There are several similar problems given. The goal is to assist students in constructing the calculus material. In this case the computer can be used as a learning tool, where students perform mental constructions: actions, processes, objects, and schemes to understand the concept (Arnawa, 2009).

Table 1 below is an example of Maple V11 command about counting Riemann sums. The command was typed by the student and then executed in the Maple application program. The result is copied in the place provided.

<table>
<thead>
<tr>
<th>NO</th>
<th>MAPLE command</th>
<th>MAPLE answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>g:= x -&gt; 2*x^2-8; a:= -1; b:=3;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>middlebox(g(x),x=-1..3,8);</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Delta := (b-a)/n;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x[k] := a + k*Delta;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sum(g(x[k])*Delta,k=1..n): % = simplify(value(%));</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Limit(%%,n = infinity): %= evalf(value(%));</td>
<td></td>
</tr>
</tbody>
</table>

To help students understand the results of the execution of Maple, then in the small group discussion phase provided the questions as in Figure 1. Continued by calculating the problem manually. Here's an example of a question about Maple's answer in the Small Group Discussion phase.

Perhatikanlah jawaban Maple untuk kegiatan no 4 pada Tabel 1 di atas.

1. Titik apa yang menjadi titik sampel pada masing-masing poligon?
2. Berapa nilai fungsi untuk masing-masing titik sampel?
3. Salinlah jawaban Maple untuk gambar fungsi g pada kegiatan no 4 kemudian hitunglah secara manual berapa luas poligon tersebut? (ikutilah langkahkah Maple tsb untuk menghitungnya).
4. Bagaimanakah caranya menentukan lebar partisi pada gambar? Berikan contohnya
5. Jika x_0 = a, dan x_n = b, delta = (b – a)/n. Tentukanlah x_1, x_2, ... dan x_k untuk fungsi g
6. Terdapat perintah Sum... untuk apa perintah Sum ... itu? berikan contohnya
7. Jelaskanlah dengan ringkas, mengapa ada perintah Limit(%%,n = infinity): % = evalf(value(%));... ?
8. Hitunglah berapa Jumlah Riemann untuk n = ∞?
9. Jelaskanlah dengan ringkas, kapan hasil jumlah Riemann adalah luas daerah yang sebenarnya (luas daerah di bawah fungsi f yang dibatasi oleh x = a, dan x = b)?

The next step was to discuss the answers to the questions given manually ie without the help of a computer. Solving these problems in the Classroom Discussion phase should be explained by the group selected in front of the class. After the class discussion time runs out, then proceed with the Exercise phase. Usually a little more time left, so it's a matter of choosing a short answer which can be
completed in class, then the remaining exercise on the Worksheet for Exercise phase, completed at home by each student as homework.

When the computer was unable to provide explanations to the students about a material, then the lecturer task to provide scaffolding (assistance). Assistance can be given to individuals, to groups, can also be classical, depending on the level of difficulty faced by students. Sometimes students look for answers by reading the source book.

III. THE RESULT OF MODEL APOS IMPLEMENTATION

To know the level of understanding of the students after APOS Model applied on the topic of calculating the area of polygon and counting the Riemann Sums, the postes were executed with the following questions:

<table>
<thead>
<tr>
<th>SOAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Lukislah daerah dibawah grafik fungsi Y = -X² + 9 untuk a = 0 dan b = 2, dan dipartisi sebanyak n = 8</td>
</tr>
<tr>
<td>2. Hitunglah luas poligon soal nomor 1 menggunakan (pilih salah satu sesuai nomor ):</td>
</tr>
<tr>
<td>A. Poligon luar</td>
</tr>
<tr>
<td>B. Poligon dalam</td>
</tr>
<tr>
<td>C. Poligon dengan titik sampel = titik tengah</td>
</tr>
<tr>
<td>3. Hitunglah soal no 1 untuk n = tak hingga</td>
</tr>
</tbody>
</table>

After the students' answers were checked, the following info was obtained: The number of students who scored ≥ 80 was 33.33%; The number of students who scored ≥ 60 and <80 were 33.33%; The number of students who scored <60 was 09%. Before the postes begun, students were asked to answer an open questionnaire to find out in which phase were the new students can understand the material for learning using the APOS Model, and explained what the reason. From the 38 students, two students were absent, so the answer was 36 people. From the calculation results were obtained that the number of students who just start to understand the material in the phase: (1) Practicum phase = 22.22%, (2) small group discussion phase = 30, 56%, phase Discussion Class = 33, 33%, and exercise phase = 13, 09%.

Some reasons why students choose in the phase where they just can understand the material, in the learning with APOS model, where students have to construct their own materials he studied, were as follows:

X31 student code
"At practicum phase, I feel faster understand and very happy in learning, I can immediately try and feel challenged to use maple application. From the practice (homework) I can practice to solve the problem, so if there are problems with different questions I can solve them "

Student code X19
"I understand the material when starting the group discussion. When practicum we just put the notations into the maple application and instantly get the answer. In that phase we just write what the maple answer showed to us. But when the group discussion I just understand how the answer with the manual way. When class discussions there were time to correct whether what we discuss in the group is correct or not so it will be more convincing our answers."

Student code X29
"I began to understand during the small group discussion, because at that time our group solved the problem. In solving the problems, we could exchange ideas or ask each other if we did not understand about the material discussed. Because the group is a friend itself, it was more comfortable to asked questions without feel ashamed"

X9 student code
"I just understood in class discussions because in class discussions I can understand how the steps and how to solve in detail, because there was a group coming forward to explain a problem with a detailed solution. I can understood in the group discussion phase because in this group we can be mutually communicate and helping each other in terms of learning the problems that have been given. I also understood in the exercise (homework) phase because I have learn in the group discussion and class"

Student code X15
"I just understand the material in the excerise phase (HW) because in that phase I try to find a way/solution of the existing problem. HW work is also done at home in a state of calm, so comfortable for learning without any disturbance from others. In the pre-exercise phase the material being studied has not been fully understood, although I am a little bit has begun to get them"

IV. DISCUSSION
Based on the results of the poll about on what phase where the students can understand the
material about the area of polygons and the sum of Riemann, obtained the info that there were 22% students who began to understand the material in the practical phase. This students were helping to explain the material to his friends in small groups or on class discussions. It is seen that there are 30, 56%, students can understand the material through small group discussion, and after class discussion there are 33.33% who can already understand the material. The rest can only understand the material after doing homework.

In general, students said that they can understand the material when they are discussing with small group friends or when other groups present their work in class. Some of them were asking directly to the lecturers, and some groups asked the assistants in each meeting. This showed that there has been a multidirectional communication from students to lecturers or assistants, and from student to student. This communication is in the framework of providing assistance by lecturers and assistants to students or by students to students. This is in accordance with the following Slavin opinion

“Cooperative learning comprises "instructional methods in which teachers organize students into small groups, which then work together to help one another learn academic content". Cooperative learning consists of five basic elements: positive interdependence, promotive interaction, individual accountability, teaching of interpersonal and social skills, and quality of group processing. Learning situations are not cooperative if students are arranged into groups without positive interdependence (Johnson & Johnson). Positive interdependence means that in cooperative learning situations, students are required to work together as a cohesive group to achieve shared learning objectives (Yager). In the process, students must be responsible for the other group members’ learning” (Slavin) in (Tran, Van Dat, 2014).

Key ratios of successful (productive) educational process are skills of thinking, working with information (search, analyse, select, estimate), solving creatively cognitive and practical tasks, competence in solving the problems independently, skills to realize reflection of activities, to state precisely one’s thoughts etc (Cunska, Aija dan Savicka, Inga. 2012).

The following conditions of organisation of interactive learning should be obligatory in lessons of Maths:

- Relations between the teacher and the students should be based on mutual confidence and be positive;
- Style of teaching should be democratic;
- The process of co-operation between the teacher and the students or among the students should be observed;
- Everything should be based on students’ personal mathematical experience, striking examples, facts and examples should be included;
- Introductory tasks should be included as well as there should be enough time for doing basic tasks;
- The used methods should correspond to students’ age;
- Outer and inner motivations of activities as well as students’ mutual motivation should be included;
- Speed and abilities of every student should be taken into account;
- Reflexive ties should be created by doing express-questionnaires, promoting discussions.

Several teachers in their researches have acknowledged that effectiveness (in per cents) of acquiring the learning material depends on the correct use of the method: lectures (5%), reading (10%), use of video un audio materials (20%), demonstrations (30%), group discussions (50%), practical activities (75%), peer-teaching and immediate use of knowledge in practice (90%) (Cunska, Aija dan Savicka, Inga. 2012). It was described as a learning experiences cone in Figure 3.
Figure 3 shows that if the teacher presents the material, accompanied by a picture or diagram showing, or the teacher plays a video or film, even if the teacher demonstrates it then the student remembers ≤ 30%, the student tends to passively. When students engage in discussions, presenting what they learn that students can remember ≥ 50%, students will be active in learning. Based on the results of the polls and posttest results proved that when students engaged in the discussion, presenting what they learned that can be remembered students ≥ 50%, they also will be active in learning.

According to Yunus Abidin (2016), learning contains two main characteristics those are: (1) the learning process involves the students’ mental processes maximally which want the students’ activity to think and (2) the learning is directed to improve and improve students’ thinking ability which in turn thinking activities can help students to gain knowledge of their own construction. Based on these two characteristics clearly learning is not only done as a transfer of knowledge but activities that must be actively undertaken in an effort to build students own knowledge based on the potential they has. It can be concluded that the Mathematical Learning Model Based on APOS Theory (APOS Model) is including the Learning Model in the context of the 2013 curriculum.

Use of ICT in lesson:

- Increases students motivation for learning (not only because it is up-to-date and innovative – it helps to choose and differentiate tasks according to students’ knowledge and skills);
- Helps students to finish the tasks during the lesson because teacher and technology resources help them;
- Helps students to understand practical meaning of mathematical tasks. For example, it is very easy with the help of computer programs to change parameters of the given values and model different results;
- Helps the teacher to make educational process more individual and differentiated because of the interactive dialogue between a student and a computer in the speed and place suitable for the student;
- Educational process becomes more visual, colourful and attractive;
- Creates feeling of comfort for students (they can learn in their own speed and cooperate with other students) and teachers (they can operate only with those students who need teacher’s help);
- Helps teachers to oversee the class and lead the educational process successfully;
- Does not allow subjective evaluation of students’ knowledge because the mark is given by the program according to the number of correct answers;
- Helps students to develop their own knowledge according to the marks they have got – students can repeat solving the tasks, can ask help to the teacher, analyse their own mistakes;
- Develops skills of self-management and self-assessment in students;
- Helps students to solve dilemma – from one side: a great amount of learning material should be acquired in limited period of time – from other side: for a successful learning process inclass different methodical materials should be chosen very carefully;
- Helps students to rise “pedagogical mastery”- pedagogical competences,
specific knowledge and skills, characteristic traits what help the teacher to lead the educational process effectively (Cunska, Aija dan Savicka, Inga. 2012)

Students who generally used ICT had better results than those who did not use ICT. Furthermore, students who use ICT in their education showed better results than students whose use of ICT is not related to education (Kubiatko & Vickova.)

CONCLUSION

Computer based APOS Model in mathematics learning is a Learning Model in the context of the 2013 curriculum, with syntax consisting of phases: Orientation, Practicum, Group Discussion, Class Discussion, Exercise, and Evaluation. In the Practicum phase, students work using Laptops with the Maple V11 application program. To support the implementation of the APOS Model, a workspace was developed which included activities in each phase.

Implementation of APOS Model in Integral Calculus Course at Student 3rd Semester of Mathematics Education Study Program FKIP UNIB FY 2017/2018 has given info that: number of freshmen at Phase: Practicum amounted to 22.22%, Small Group Discussion Phase = 30, 56%, Classroom Discussion Phase = 33, 33%, Exercise Phase = 13, 09%. : The number of students who scored ≥ 80 was 33,33%; The number of students who scored ≥ 60 and <80 were 33,33%; The number of students who scored <60 was 33,33%.

It can be conclude that computer based APOS Model is very good in developping college students learning outcomes, and to develop students discussions skill.

REFERENCES

Books :


Documents from internet:


