PROCEEDINGS

2016 12th International Conference on Mathematics, Statistics, and Their Applications (ICMSA)

In Conjunction with The 6th Annual International Conference of Syiah Kuala University

4 - 6 October 2016
AAC Dayan Dawood, Syiah Kuala University
Banda Aceh, Indonesia

ANNUAL INTERNATIONAL CONFERENCE AIC 2016 UNIVERSITAS SYIAH KUALA

IMT-GT
The 12th ICMSA 2016

Organized by:
Department of Mathematics
Faculty of Mathematics and Natural Sciences
Syiah Kuala University

AIC - ICMSA 2016 Committees

Advisory Committees:

Prof. Dr. Ir. Samsul Rizal, M. Eng., Rector of Syiah Kuala University
Dr. Hizir Sofyan, Vice Rector for Academic Affairs of Syiah Kuala University
Prof. Dr. Ir. Hasanuddin, M. S., Head of Institution of Research and Community Service, Syiah Kuala University
Nurmaulidar, M. Sc., Head of Department of Mathematics, Syiah Kuala University

Organizing Committees:

Chairman:
Dr. Syaifullah Muhammad, M. Eng.

Vice Chairman:
Prof. Dr. Samadi, M. Sc.
Dr. Rini Oktavia, M. Si., M. A.

Secretary:
Dr. Nasrul Arahman, M. T.
drh. Triva Murtina Lubis, M. P.

Treasurer:
Dr. Chenny Seftarita, SE, M. Si.

International Scientific Committees:

Prof. Dr. Herman Mawengkang, University of North Sumatera, Indonesia
Prof. Dr. Dato' Rosihan Ali, Universiti Sains Malaysia, Malaysia
Prof. Dr. Anton Abdul Basah Kamil, Universiti Sains Malaysia, Malaysia
Dr. Hizir Sofyan, Syiah Kuala University, Indonesia
Prof. Dr. Saib Suwilo, University of North Sumatera, Indonesia
Dr. Tarmizi Usman, Syiah Kuala University, Indonesia
Prof. Dr. Engku Muhammad Nazri Engku Abu Bakar, Univ. Utara Malaysia, Malaysia
Dr. Suhaida binti Abdullah, Universiti Utara Malaysia, Malaysia
Assoc. Prof. Dr. Putipong Bookkamana, Chiang Mai University, Thailand
Assoc. Prof. Dr. Surapong Auwatanamongkol, NIDA, Thailand
Asst. Prof. Dr. Winai Bodhisuwan, Kasetsart University, Thailand
Assoc. Prof. Praiboon Pantaragphong, KMITL, Thailand
Prof. Dr. Suthep Suantai, Chiang Mai University, Thailand
Prof. Samruam Chongcharoen, NIDA, Thailand

Local Technical Program Committees (TPC):

Dr. Salmawaty Arif, M. Sc.
Dr. Said Munzir, M. Eng. Sc.
Dr. Taufik Fuadi Abidin, M. Tech.
Dr. Zurnila Marli Kesuma, M. Si.
Dr. Rahmah Johar, M. Pd.
Dr. M. Ikhsan, M. Pd.
Muslim Amiren, M. Info. Tech.
Mahmudi, M. Si.
Nunung Mardatilla, S. T.
Muhammad Chandra Gunawan, A. Md.
Devi Firmansyah, S. Si.

Editors:

Dr. Salmawaty Arif, M. Sc.
Dr. Rini Oktavia, M. Si., M. A.

Reviewers:

Dr. M. Shabri Abdul Majid, Syiah Kuala University, Indonesia
Dr. Suhaida Abdullah, Univerisiti Utara Malaysia, Malaysia
Dr. Taufik Fuadi Abidin, Syiah Kuala University, Indonesia
Ms. Hafnani Ahmad, Syiah Kuala University, Indonesia
Mr. Saiful Amri, Syiah Kuala University, Indonesia
Dr. Salmawaty Arif, Syiah Kuala University, Indonesia
Dr. Fitri Arnia, Syiah Kuala University, Indonesia
Prof. Dr. Hasmawati Basir, Hasanuddin University, Indonesia
Prof. Dr. Edy Tri Baskoro, Bandung Institute of Technology, Indonesia
Dr. Winai Bodhisuwan, Kasetsart University, Thailand
Prof. Dr. Edi Cahyono, Haluoleo University, Indonesia
Dr. Yosza Dasril, Universiti Teknikal Malaysia Melaka, Malaysia
Dr. Rianto Ahmadi Djojosugito, Society of Actuaries of Indonesia, Indonesia
Prof. Dr. Engku Muhammad Nazri Engku Abu Bakar, Univ. Utara Malaysia, Malaysia
Dr. Gan Fatt, National University of Singapore, Singapore
Dr. Taufiq A. Gani, Syiah Kuala University, Indonesia
Mr. Agah Garnadi, Institut Pertanian Bogor, Indonesia
Dr. Muhammad Ikhsan, Syiah Kuala University, Indonesia
Prof. Dr. Seok-Oh Jeong, Seoul National University, Korea
Dr. Rahmah Johar, Syiah Kuala University, Indonesia
Dr. Zurnila Marli Kesuma, Syiah Kuala University, Indonesia
Dr. Tiffany Kolba, Valparaiso University, USA
Dr. Wuryansari Kusumawinahyu, Brawijaya University, Indonesia
Dr. Son Kuswadi, Politeknik Elektronika Negeri Surabaya, Indonesia
Dr. Saiful Mahdi, Syiah Kuala University, Indonesia
Dr. Miftahuddin, Syiah Kuala University, Indonesia
Dr. Intan Muchtadi-Alamsyah, Bandung Institute of Technology, Indonesia
Dr. Said Munzir, Syiah Kuala University, Indonesia

Dr. I Wayan Mustika, Gadjah Mada University, Indonesia
Dr. Nizamuddin, Syiah Kuala University, Indonesia
Dr. Endar Nugrahani, Bogor Institute of Agriculture, Indonesia
Prof. Dr. Toto Nusantara, Universitas Negeri Malang, Indonesia
Dr. Rini Oktavia, Syiah Kuala University, Indonesia
Dr. Dedy D. Prastyo, Sepuluh Nopember Institute of Technology, Indonesia
Prof. Dr. Lara Pudwell, Valparaiso University, USA
Dr. Purhadi, Sepuluh Nopember Institute of Technology, Indonesia
Dr. Santi W. Purnami, Sepuluh Nopember Institute of Technology, Indonesia
Prof. Dr. Marwan Ramli, Syiah Kuala University, Indonesia
Prof. Dr. Dedi Rosadi, Gadjah Mada University, Indonesia
Dr. Rizky Rosjanuardi, Universitas Pendidikan Indonesia, Indonesia
Dr. Siti Rusdiana, Syiah Kuala University, Indonesia
Dr. Masimin Sanmurja, Syiah Kuala University, Indonesia
Dr. Hizir Sofyan, Syiah Kuala University, Indonesia
Prof. Dr. M. Alejandra Sorto, Texas State University, USA
Dr. Muhammad Subianto, Syiah Kuala University, Indonesia
Dr. Novriana Sumarti, Institut Teknologi Bandung, Indonesia
Prof. Dr. Agus Suryanto, Brawijaya University, Indonesia
Dr. Intan Syahrini, Syiah Kuala University, Indonesia
Prof. Dr. Tulus, University of North Sumatera, Indonesia
Dr. Muksin Umar, Syiah Kuala University, Indonesia
Prof. Dr. Alexander White, Texas State University, USA
Prof. Basuki Widodo, Sepuluh Nopember Institute of Technology, Indonesia
Prof. Widowati, Diponegoro University, Indonesia
Dr. Zulfakhriza Zulhan, Bandung Institute of Technology, Indonesia

TABLE OF CONTENTS

Technical Session on Mathematics, Statistics, and Applications

Technical Session on Mathematics Education

The 12th IMT-GT International Conference on Mathematics, Statistics and Their Applications 2016

Forecasting Model of Frequency and Average Earthquake Strength in Bengkulu Province Based on Autoregressive Fractionally Integrated Moving Average (ARFIMA) Approach

Jose Rizal[a1,*], Fachri Faisal[a2], Suhendra [b1] and Nanang Sugianto [b2]

[a12]*Mathematics Department, Mathematics and Nautral Sciences Faculty, Bengkulu, Indonesia*
[b12]*Physics Department, Mathematic and Natura; Science Faculty, Bengkulu, Indonesia*

Abstract

The aim of this research was made forecasting earthquake model in Bengkulu province areas. It was included to identify earthquake area, frequency of earthquake every month, and average of earthquake magnitude for every month. They have obtained by two analytical approach, they were descriptive statistical and inference statistical analysis that included in Autoregressive Fractionally Integrated Moving Average (ARFIMA). Variable in this research was the frequency of earthquakes in one of month (A) and average of earthquake magnitude in one of month (B). Data processing was used historical earthquake from January 2000 until May 2016. *R 3.02 Software, Oxmetrics 4, ArcGis 10.2, dan Surfer 13* were some software applications that used in processing and interpretation. The result of this results showed that the best forecasting model (criteria in sample dan out sample minimum) for frequency of earthquake was ARFIMA (2, 0.633, 1) and for magnitude of earthquake (SR scale) ARFIMA model were (1, 0.856, 1). Analysis result showed that the frequency of earthquake for three month future would be 21 times for Juny, 21 times for July, and 19 times for agustus 2016. The magnitude of earthquake for three month future were estimated about (in SR scale) 3.25, 3.23, dan 3.23. The number frequency of earthquake in province Bengkulu going to be high at near of Seluma, Bengkulu City, Central Bengkulu and North Bengkulu districts.

Keywords: Forecasting earthquake, number of frequency, earthquake magnitude, ARFIMA

1. Introduction

Earthquake was be an interesting topic that have gone by every interdisciplinary like Spatial statistics, mathematics model and geophysics. Characteristic of Indonesia's earthquake have been analysed [1], they have analysis the seismic data with apoint Process approach. In the same case but used space-time approach to the epidemic for the region Java and Bali [2]. In the next result model of the earthquake in Northwestern Sumatra with Markov model approach [3]. Besides the approach mentioned above, there are many other approaches in modeling the data that depend to a certain time unit. As for the model in question time series [4] model include *Autoregressive* (AR), *Moving Average* (MA), *Eksponential Smoothing*, *Autoregressive Integrated Moving Average* (ARIMA), *Seseonal Autoregressive Integrated Moving Average* (SARIMA), dan *Autoregressive Fractionally Integrated Moving Average* (ARFIMA).

Each method has drawbacks and advantages. The researchers have compared the results of ARIMA forecasting models and ARFIMA models with the index case study air pollution that occurred in the Shah Alam Selangor. The conclusion of them, by looking at the minimal value of MAPE, ARFIMA models provide better forecasting results than ARIMA model [5]. In addition, still in the same post, the ARFIMA models able to cope when the data has a long memory properties.

As a beginner result, we have studied the incidence of earthquake in Sumatera Island. It was only given quantity information of earthquake happened. The blue was magnitude $\leq$ 4.85 SR and the red was showed magnitude > 4.85 SR. in other side, it was showed a ratio between the actual data with the forecasting result [6].

We were used similar method with older research, but the research area was replaced with Bengkulu province as one of province in Sumatera Island. Adding parameter in this result, we were analysed the pattern of epicentre location.

A pattern of epicentre location, number of frequency, and magnitude, they were be principle to get more comprehensive and more precision forecasting model of earthquake in Bengkulu Province. Based on background above, can be formulated which is the goal of this research was to building a forecasting model of frequency number of earthquake in Bengkulu Province. Three models would be explain of those models were to identification of earthquake area, the frequency of earthquake in one month and average of earthquake magnitude for one month.

* Corresponding author. HP.081321420921.
E-mail address: Jrizal04@gmail.com.

2. Material and Method

ARFIMA model have three parameters, included *Autoregressive (p)*, *Moving Average (d)*, and *d* was differencing parameter. ARFIMA model ($\boldsymbol{p}, \boldsymbol{d}, \boldsymbol{q}$) has developed by Granger and Joyeux (1980) as given below Equation [7], [8].

$$\phi(B)\nabla^d Z_t = \theta(B)e_t \quad (1)$$

The ARFIMA models able to cope when the data has a long memory properties, examination of *Long Memory* could be done by looking at *Hurst* value based on statistics of *R/S*.

$$R/S = c.t^H, \quad (2)$$

if $H = 0{,}5$, characteristic of time series data was *short memory*. The intermediate memory if $0 < H < 0{,}5$ and *long memory* for $0{,}5 < H < 1$. [7], [9]

The best ARFIMA Model was a model which showed that estimation of their parameter was the most significant. If θ was a parameter in ARFIMA model, $\hat{\theta}$ was be a estimated value from those parameter and $\mathrm{E}(\hat{\theta})$ was a *standard error* of $\hat{\theta}$, so the examination of signify parameter could be estimated by hypothesis H_0: $\theta = 0$ vs H_1: $\theta \neq 0$, Conclusion: H_0 rejected if $|t| > t_{\alpha/2, df=n-n_p}$ or *p-value* < α. Tests of normality can be calculated by using the *Kolmogorov-Smirnov*, if *p-value* < α then H_0(residual normal distribution) value would be rejected. Examination of *White Noise* test conducted using the Ljung-Box test, decision-making criteria: H_0 (*White Noise*) rejected if *p-value* < α.

Selecting the Best ARFIMA Model Criteria that used at *in sample* was AIC (*Akaike's Information Criterion*), where the best model choices by development of minimal AIC value. Necessary parameters including in this calculation were the number of observation (n), maximum likelihood estimation ($\hat{\sigma}_a^2$), and the number of ARMA parameter (M), as given by [4],[8]:

$$AIC(M) = n \ln \hat{\sigma}_a^2 + 2M \quad (3)$$

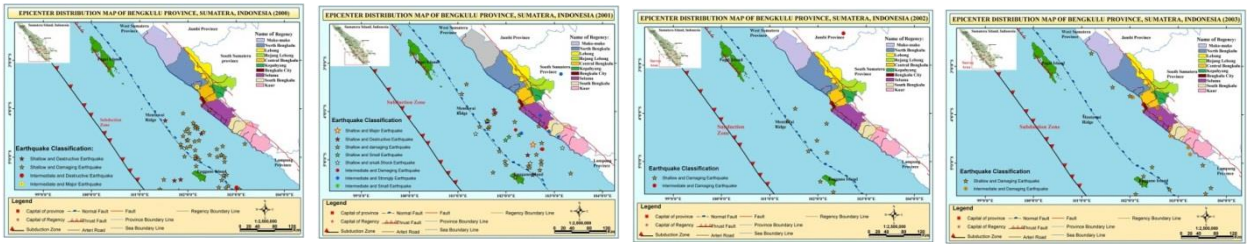
Criteria that used at *out sample* was RMSE (*Root Mean Square Error*), as given by:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Z_t - \hat{Z}_t\right)^2} \quad (4)$$

3. Results and Discussion

3.1. Descriptive analysis of Earthquake

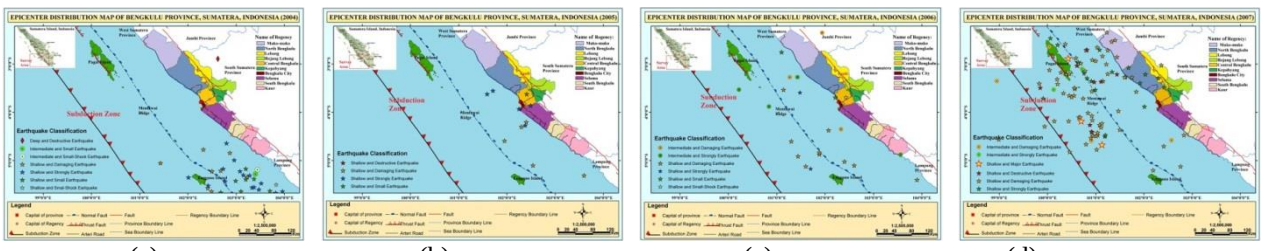
For simplicity in writing, let's say the variable A is the frequency of earthquakes within a month and the variable B represents the average (monthly) of an earthquake. it can be described, earthquakes that occurred in Bengkulu from year 2000 to 2016, every month an earthquake of at least 1 times and at most 114 times.



(a) (b) (c) (d)

Fig. 1. Earthquake epicentre map at (a) 2000; (b) 2001; (c) 2002; (d) 2003

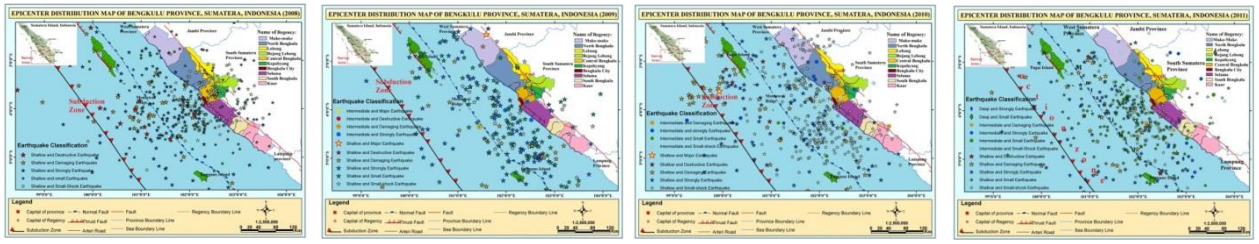
To see the movement patterns of earthquakes each year, used the help Sotware 133 applications Surfer and ArcGIS 10.2. Classification as a differentiator Earthquake, different markings were made for eight classification of earthquakes (can be seen in each image). Seen in Fig. 1 (a), throughout 2000, earthquakes occurred in the city of Bengkulu, Seluma, South Bengkulu Regency, and Regency of Kaur. Still in the same year, there were four times with classification Shallow earthquakes and Destructive Earthquake.



(a) (b) (c) (d)

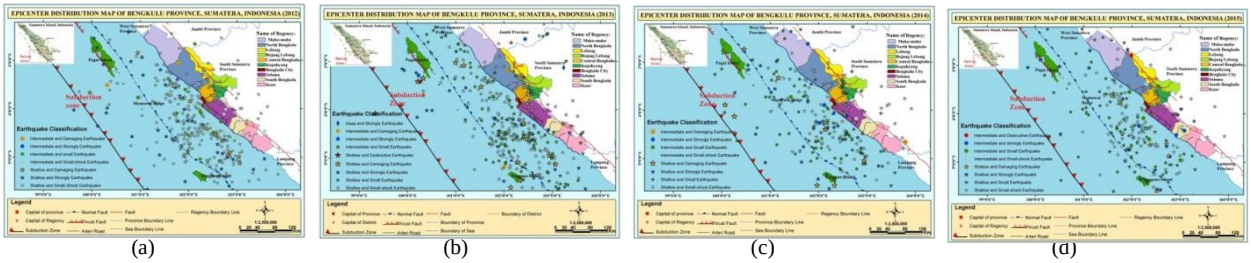
Fig. 2. Earthquake epicentre map at (a) 2004; (b) 2005; (c) 2006; (d) 2007

During 2001 (Fig.1 (b)), the distribution pattern of earthquake occurrence is relatively dispersed, only Mukomuko who do not experience earthquakes. In that year, there were three times with classification Shallow earthquakes and Destructive Earthquake. In 2002 and 2003 (Fig. 1 (c),(d)), earthquakes are relatively minimum. In 2004 (Fig. 2. (a)), earthquakes tend to cluster towards the south of Bengkulu province, while the districts that experienced earthquakes were South Bengkulu and Kaur.



(a) (b) (c) (d)

Fig. 3. Earthquake epicentre map at (a) 2008; (b) 2009; (c) 2010; (d) 2011



(a) (b) (c) (d)

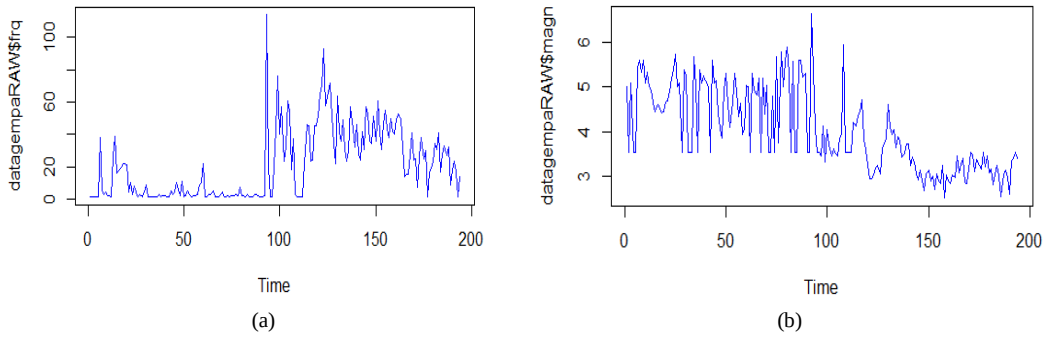
Fig. 4. Earthquake epicentre map at (a) 2012; (b) 2013; (c) 2014; (d) 2015

Back in 2005 and 2006 (Fig. 2. (b), (c)), earthquakes relative minimum and spread. In 2007 (Fig. 2. (d)), earthquakes tend to cluster to the north of Bengkulu province, there were several earthquakes Shallow and Destructive Earthquake. In 2008 (Fig. 3 (a)), earthquakes tends to happen in the middle of Bengkulu province and dominated earthquakes and Strongly Shallow Earthquake. From 2009 to 2015 (Fig. 3 and Fig. 4), the pattern of earthquake occurrence is spread evenly in Bengkulu. Based on the pattern of epicenter movement on each year, number frequency of earthquake in province Bengkulu will be high at near of Seluma, Bengkulu City, Central Bengkulu and North Bengkulu districts. In general, Epicenters were distributed at Subduction zone, especially in the Mentawai ridge of Muko-Muko to Kaur districts.

The center of epicenters not only occur in ocean or caused by the movement of oceanic plates to continental plates but also had been in fault zones or it was called ground earthquake. Their Magnitude has relatively smaller than the earthquake in the ocean. Average they have occurred at shallow depths and micro to small earthquakes of magnitude. This phenomenon can be seen earthquakes in 2008 and 2015. An earthquake of this land did not have the significant effect, but if the strength of the earthquake reached 4 to more, it going to be very dangerous an earthquake for the surrounding societies.

3.2. Modelling Frequency and Strength Earthquake with ARIMA Approach

We have used a publication about *R package* for ARFIMA Model [10]. There were some procedure to estimated ARFIMA model parameters. The first step was saw plot from each varables. Based on Fig. 5 (a) and (b), they were estimated that variable A dan B have stationary in average, because their trend were tend to be flat.



(a) (b)

Fig. 5. Graphic monthly for 2000 – 2016 (a) Variable A; (b) Variable B;

In other side, two pictures also available some data that dangle either up or down indicating both the data is not stationary in variants. Of course, it was needed next verification.

Table 1. The result of stationary examination in averages and variant of A and B variables

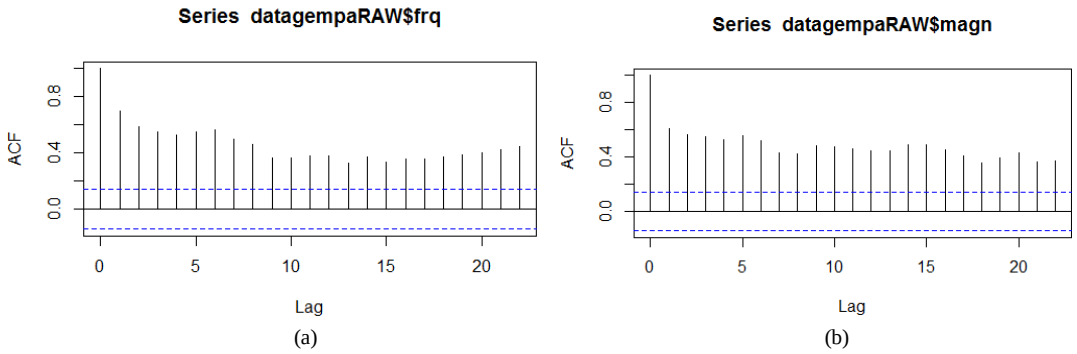
Variable	Testing Stationarity In average			Testing Stationarity in variant		
	ADF (t_δ)	*p-value*	Result	λ	The result of test	$T(Z_t)$
A	-76,597	0,010	stasionary	0,110	Not stasionary	$Ln\ (Z_t)$
B	-156,38	0,010	stasionary	-0,999	Not stasionary	$1/Z_t$

According table 1, variable of A and B have been stationary when average, but they no stasioner in variant. We transform the data for each of the variables A ($Ln\ (Z_t)$) and B ($1/Z_t$). By testing the variant stationary in return, obtained for each variable value $\lambda = 1$. This shows the data has been stationary in variants. Based on this, we could continued on to testing long memory.

At Fig. 6 (a) and (b), showed that autocorrelation of them for every *lag something increase and decrease and relatively not convergent to zero*, so that identify of *long memory* using ACF pattern approach more difficult to gone. According by get equation 2, we could to do that by get *Hurst* (*H*) value from statistical of R/S. *Hurst* value for variable A dan B were 0,803 and 0,805 respectively. Because *Hurst* value of each variable have interval between 0,5 until 1, we conclude that data on each variable have *long memory* characteristic. If characteristic of data was long memory, we need determine fractional differencing value (*d*).. In Table 2, we could choosen *d* value for variable A between 0.391 to 0.875. As for variable B between 0.614 to 1.098.

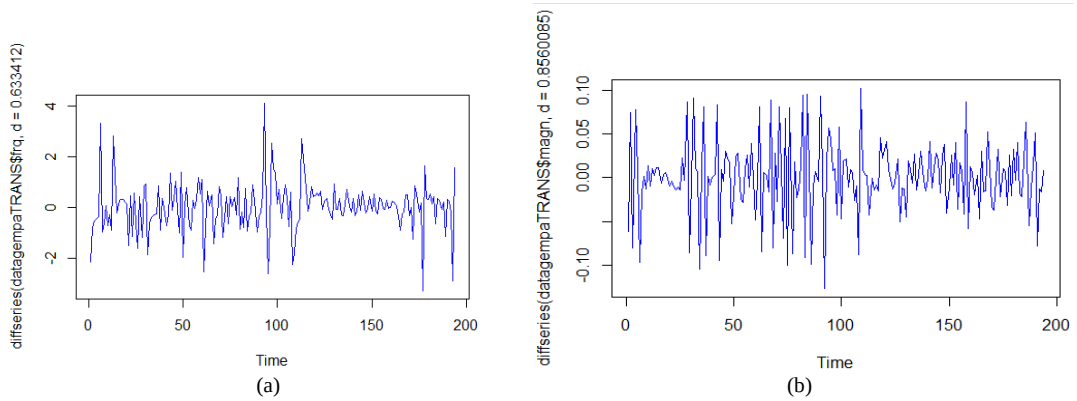
Table 2. Estimation of differencing fractional (*d*) for Variable A and B

Variable	d-Value	Standard Deviation	
		Asymptotic	Error
A	0,633	0,242	0,267
B	0,856	0,242	0,192



(a) (b)

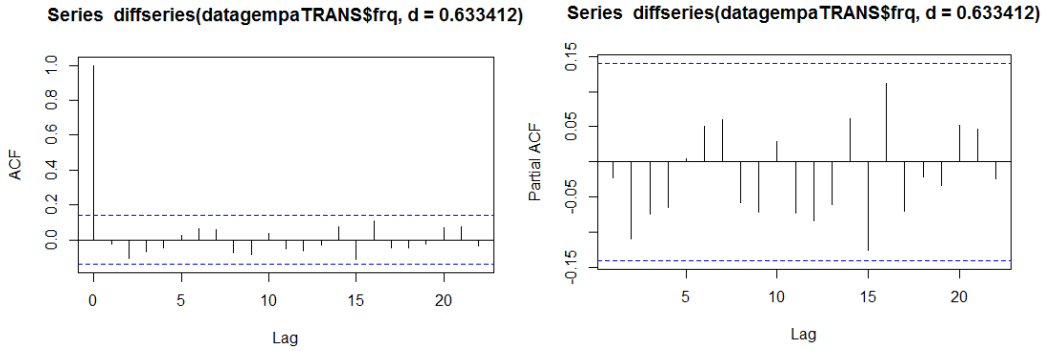
Fig. 6. Graph of ACF (a) Variable A ; (b) Variable B;



(a) (b)

Fig. 7 Graphic monthly for 2000 – 2016 have been differentiated (a) Variable A; (b) Variable B;

We choose the value of *d* for each variable according to the default of the recommendation *R*. Data has been transformed to Box-Cox and differencing would have some properties, like as stationary with average, stationary with variant, and didn't have long memory properties, as shown at Fig. 7 (a) and (b). The first step in determining the value of the model parameters, we need to considerred the characteristics of ACF and PACF patterns to lag. For variable A, because the plot ACF has exponential decay and there is no information obtained from PACF plot, allegedly while the model were *Autoregressive* models (p) (Fig. 8 (a) and (b).

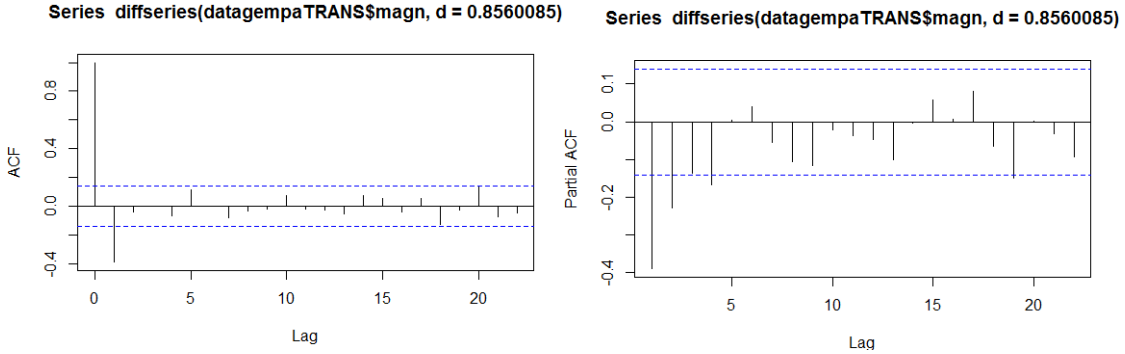


(a) (b)

Fig. 8 Graph from variable A that have been differentiated (*d*=0,633) (a) ACF; (b) PACF;

As for the variable B, ACF and PACF plot exponential decay at lag 1, while the models were *Autoregressive* (1) and *Moving Average* (1). We could get the optimal parameters to obtain optimal parameters, can be done by trial and error around the value of the order was elected (Fig.9. (a) and (b)).

The election kiteria order AR and MA optimal based on criteria in the sample and the sample out (equation (3) and (4)). Based on the results of significance test parameters in Table 3, it could be the best time series models for variable A was a ARFIMA model (2,0.633,1), while for the variable B was ARFIMA model (1,0.856,1).



(a) (b)

Fig. 9 Graph from variable B that have been differentiated (*d*=0,856) (a) ACF; (b) PACF;

Table 3. Model parameter tests of Variable A dan B

Variable	Model	Parameter	Coefisien	p-value	Result	RMSE	AIC
A	ARFIMA (2,0.633,1)	AR 1	1,282	0,000	Significant	4,44	8,245
		AR 2	-0,289	0,010	Significant		
		MA 1	-0,816	0,000	Significant		
B	ARFIMA (1,0.856,1)	C	4,420	0,000	Significant	0,842	2,049
		AR 1	0,990	0,000	Significant		
		MA 1	-0,8160	0.000	Significant		

The next step were examination of white noise for some lag (Table 4), residual of models have distributed normal for each different lag. Then, the result of ARCH LM test shown that residual of ARFIMA model (2, 0.633, 1) was not autocorrelation and residual of ARFIMA model (1,0.856,1) was autocorrelation. This result has affected a forecasting quality.

Table 4. Ljung-Box Residual and ARCH LM Test

Variable	Model	Ljung-Box Test Residuals (White Noise)			ARCH LM Tests		
		Lag	*P-value*	Result	Lag	*P-value*	Result
A	ARFIMA (2,0.633,1)	1	0,978	Normal	2	0,575	No autocorrelation
		8	1.000	Normal	5	0,894	No autocorrelation
		14	0,451	Normal	10	0,815	No autocorrelation
B	ARFIMA (1,0.856,1)	1	0,292	Normal	2	0.000	Autocorrelation available
		5	0,997	Normal	5	0.000	Autocorrelation available
		9	0,389	Normal	10	0.001	Autocorrelation available

By using equation (1), ARIMA (2,0.633,1) model of variable A, could written in the form of the following equation:

$$\phi_2(B)\nabla^{0,633}Z_t = \theta_1(B)e_t$$

$$(1 - 1{,}282B + 0{,}289B^2)(1 - B)^{0,633}Z_t = (1 - 0{,}816B)e_t$$

While ARIMA (1,0.856,1) for variable B, can be written as follows:

$$\phi_1(B)\nabla^{0,856}Z_t = \theta_1(B)e_t$$
$$(1-0{,}990B)(1-B)^{0,856}Z_t = (1-0{,}816B)e_t$$

The results of forecasting the next three periods (July, August, and September 2016): 21 times, 21 times and 19 times. The results of forecasting the next three periods (July, August, and September 2016), the magnitude of them included 3:25 SR, 3:23 SR, and 3:23SR respectively.

4. Conclusion

The conclusion of this research were the best forecasting model (criteria in sample (AIC) dan out sample minimum (RMSE)) for variabel of A (frequency of earthquake) including ARFIMA (2, 0.633, 1), for RMSE Value 4,44 dan AIC 8,245. The frequency of earthquake for three month to go (July, Agustus, and September 2016) were 21 time, 21 time, and 19 time respectively. For variabel B model, earthquake magnitude (in SR scale) were ARFIMA model (1, 0.856, 1) which were RMSE 0,842 and AIC 2,049. The magnitude that estimated for three month to go (SR scale) were 3.25, 3.23, and 3.23 respectively. The number frequency of earthquake in province Bengkulu going to be high at near of Seluma, Bengkulu City, Central Bengkulu and North Bengkulu districts.

Acknowledgements

This research has been supported by Research Ministry, Technology, and high education which Number: 042.06-0/2016 and have been done based on by Letter of Assignment Agreement Implementation Competitive Research Grant Fiscal Year 2016 Number: 896/UN30.15/LT/2016. Thanks also to Sabar Ardiansyah and all of team from BMKG Kepahyang, Bengkulu Province.

References

[1] Pratiwi H., Subanar, Danardono, and J.A.M. van der Wiede. 2008. Two-dimensional Poisson Point Process in Modeling Tectonic Earthquake in Java and Bali, Presented at The 7th World Congress in Probabilistic and Statistics, Singapore.
[2] Pratiwi H., Subanar, and Danardono. 2014. Analysis of Earthquake Data Based on Space-Time Epidemic Type Aftershock Sequence Model. International Journal of Applied Mathematics and Statistics, Int. J. Appl. Math. Stat. Vol.52 Issue No.2 ISSN 0973-1377. CESER Publications.Capurale, G.M, Skare, M. *2014. An ARFIMA-FIGARCH Analysis Long Memory in UK Real GDP 1851-2013, ISSN 1619-4535. DIW Berlin.*
[3] Darwis, S. and Rusdi. 2003. A Markov Model for the Magnitude of Earthquake in Northwestern Sumatra. J. Indonesian Math. Soc. (MIHMI), 9 (2) : 69-75
[4] Wei, W.W. 2006. *Time Series Analysis Univariate and Multivariate Methods. Second Edition.* Greg Tobin: America.
[5] Siew, L.Y., Chin, L.Y., Pauline, M.J.W. 2008. ARIMA and Integrated ARFIMA Models for Forecasting Air Pollution Index in Shah Alam, Selangor. The Malaysian Journal of Analytical Science Vol 12. No.1
[6] Rizal J., Nugroho S., Irwanto A., and Debora. 2016. Analisis Kejadian Gempa Bumi Tektonik di Wilayah Pulau Sumatera, Jurnal Matematika Vol 6, No 1 ISSN : 1693-1394. Department of Mathematics, Faculty of Mathematics, Udayana University, Kampus Bukit Jimbaran, Badung – Bali. Indonesia.
[7] Palma, W. 2007. *Long-Memory Time Series Theory and Methods*. John Wiley & Sons, Inc.: New Jersey.
[8] Box, G. E. P., Jenkins, G. N., Reinsel, G. C. 1994. *Time Series Analysis: Forecasting and Control.* Prentice Hall: New Jersey.
[9] Capurale, G.M, Skare, M. 2014. An ARFIMA-FIGARCH Analysis Long Memory in UK Real GDP 1851-2013, ISSN 1619-4535. DIW Berlin.
[10] Veenstra, J., Mcleod, A.I. 2015. The ARFIMA R Package : Exact Methods for Hyperbolic Decay Timeseries. Journal of Statistical Software Vol 23 Issue 5